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Predicting stochastic features of vapor bubble detachment in flow boiling

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Abstract—The stochastic features of vapor bubble departure and lift-off diameters in forced convection boiling are considered in terms of probability density functions (pdfs). A mechanistic model is developed for the prediction of these functions. Their prediction requires a vapor bubble detachment model and pdfs of the mean wall superheat and mean liquid velocity. The agreement with horizontal flow boiling vapor bubble departure and lift-off data is very good. To the best of the authors' knowledge, this is the first attempt at developing such a predictive capability. In order for the model to be generally applicable to flow boiling systems, the assumptions used in estimating the mean wall superheat and mean vapor velocity pdfs require further scrutiny. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The stochastic nature of the boiling process was first discussed by Fritz and Ende [1], who made quantitative measurements of vapor bubble growth rates and departure diameters in pool boiling using photographic techniques. Since this pioneering work, numerous investigators have reported stochastic variations in nucleation sites, vapor bubble growth rates and departure diameters. Some include Staniszewski [2], Strenge *et al.* [3], Han and Griffith [4], Cole and Shulman [5], Gaertner [6], Tolubinsky and Ostrovsky [7], Chekanov [8], Sultan and Judd [9, 10], Judd and Chopra [11]. Several investigators have attempted to quantify these stochastic variations. Strenge *et al.* [3] measured probability functions of vapor bubble growth rate and departure diameter for pool boiling of ether and pentane. Also, Tolubinsky and Ostrovsky [7] measured probability functions for pool boiling of water. Recently, Klausner *et al.* [12], Zeng *et al.* [13] and Bernhard [14] measured probability functions for horizontal flow boiling departure and lift-off diameters over a wide range of flow and thermal conditions using refrigerant R113.

Although it has been recognized that the stochastic features in boiling are important in predicting heat transfer rates, theoretical analyses devoted to the prediction of statistical variations in boiling processes are lacking. Strenge *et al.* [3] and Tolubinsky and Ostrovsky [7] believed that the statistical distribution of bubble departure diameters may be due to random factors associated with boiling which have not been reliably quantified through experiments. Recently, Kenning [15, 16] measured spatial distributions of wall superheat in pool boiling using a liquid crystal thermography technique. Pasamehmetoglu and Nelson [17] computed variations in pool boiling wall superheat with variations in nucleation site distribution.

This work is specifically concerned with predicting probability density functions (pdfs) of detachment diameters for pool and flow boiling. The pool and flow boiling bubble detachment models developed by Zeng *et al.* [13, 18] imply that the mean liquid velocity over the bubble, which controls the drag, and wall superheat beneath the bubble, which controls the growth rate, determines the average bubble departure and lift-off diameters. It is postulated here that the statistical variation of bubble departure and lift-off diameters observed in Klausner *et al.* [12] and Zeng *et al.* [13] is caused by the apparently randomly distributed wall superheat and the turbulent velocity fluctuations in the liquid film. In what follows, a framework for the semi-analytical prediction of bubble detachment diameter pdfs is presented. The model prediction is compared with existing R113 horizontal flow boiling probability function data. Good agreement is observed over the range of flow and thermal conditions considered (mean liquid velocity ranges from 0.3 to 0.8 m s^{-1} and mean wall superheat ranges from 8 to 16°C).

2. FORMULATION

A vapor bubble detachment model for flow boiling has been developed by Zeng *et al.* [13] which predicts that the average vapor bubble departure diameter increases with increasing mean wall superheat and decreases with increasing mean liquid velocity. Here the point of departure is taken to be the moment the bubble leaves the nucleation site, while the point of lift-off denotes the point the bubble leaves the heating surface. As shown in Fig. 1, a growing vapor bubble

NOMENCLATURE

a	vapor bubble radius [m or mm]
C_p	specific heat [J kg^{-1} C^{-1}]
d_d	vapor bubble departure diameter [m or mm]
d_L	vapor bubble lift-off diameter [m or mm]
D	inside dimension of duct [m or mm]
G	mass flux [kg m^{-2} s^{-1}]
g	gravitational acceleration [m s^{-2}]
h_{fg}	latent heat of vaporization [J kg^{-1}]
H	liquid film thickness [m or mm]
$p(\zeta)$	probability density function
q_w	wall heat flux [kW m^{-2}]
T_{sat}	saturation temperature [°C or K]
ΔT	$= T_w - T_{sat}$, wall superheat [°C]
u	local liquid velocity [m s^{-1}]
u^*	turbulent friction velocity [m s^{-1}]
u_m	area averaged velocity [m s^{-1}]
Δu	local difference between liquid flow and vapor bubble
x	vapor quality.

Greek symbols

η	liquid thermal diffusivity [m^2 s^{-1}]
θ_i	inclination angle
ν	liquid kinematic viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
$\sigma_{\Delta T}$	standard deviation of wall temperature fluctuations [°C]
σ_u	standard deviation in liquid velocity fluctuations [m s^{-1}].

Subscripts

d	departure
l	liquid
L	lift-off
v	vapor.

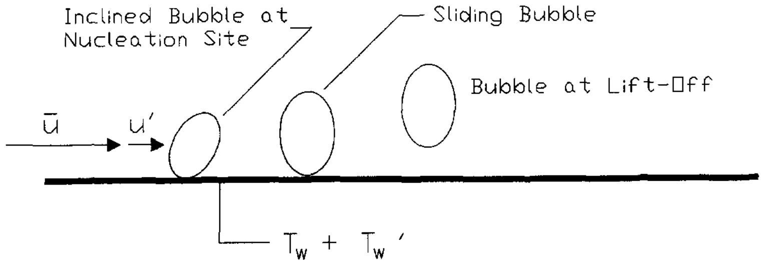

Fig. 1. Vapor bubble departure and lift-off process in horizontal flow boiling.

subjected to horizontal shear flow and heated from beneath by a solid wall will be inclined in the direction of flow. Due to the very small inertia of the vapor bubble, it will immediately respond to variations in the forces acting on it. The viscous force on the bubble tends to pull it away from the nucleation site, while the component of the growth force parallel to the heating surface tends to prevent it from departing the nucleation site. The magnitude of the growth force depends on the wall superheat beneath the bubble. Once the vapor bubble departs the nucleation site it typically slides along the heating surface until the buoyancy force is sufficient to overcome the component of growth force normal to the heating surface, at which point it lifts off. Due to the highly turbulent nature of two-phase flow, the liquid velocity passing over the bubble will vary in both time and space. In addition, due to localized cooling at nucleation centers, the wall superheat will also display spatial and temporal variations as demonstrated by Kenning [15, 16] and Pasamehmetoglu and Nelson [17]. Thus, stochastic variations in vapor bubble detachment diameters are also expected.

Based on Zeng *et al.* [13], at the point of bubble departure in horizontal flow boiling the following force balances must be satisfied

$$F_{qs} + F_{du}\sin\theta_i = 0 \quad \text{and} \quad F_b + F_{du}\cos\theta_i + F_{sL} = 0 \tag{1}$$

which may be simultaneously solved for the dependent variables, bubble radius, a, and inclination angle, θ_i. At the point of vapor bubble lift-off

$$F_b + F_{du} = 0;\quad \theta_i = 0 \tag{2}$$

which may be solved for the dependent variable, bubble radius. In equations (1) and (2), F_{qs} is the quasi-steady drag, F_{du} is the bubble growth force, F_{sL} is the shear-lift, and F_b is the buoyancy force. The following expressions have been recommended [13] for computing these forces

$$\frac{F_{qs}}{6\pi\rho_l \nu \Delta u a} = \frac{2}{3} + \left[\frac{12}{Re} + 0.75\left(1 + \frac{3.315}{Re^{1/2}}\right)\right]^{-1} \quad [19]$$

(3)

$$F_{\mathrm{du}} = -\rho_{\mathrm{l}}\pi a^2\left(\tfrac{3}{2}C_{\mathrm{s}}\dot{a}^2+a\ddot{a}\right), \quad C_{\mathrm{s}} = \tfrac{20}{3} \tag{4}$$

$$a(t) = \frac{2b}{\sqrt{\pi}}\frac{\rho_{\mathrm{l}}C_{\mathrm{pl}}\Delta T}{\rho_{\mathrm{v}}h_{\mathrm{fg}}}\sqrt{\eta t} \tag{5}$$

$$\frac{F_{\mathrm{sL}}}{\frac{1}{2}\rho_{\mathrm{l}}\Delta u^2\pi a^2} = 3.877G_{\mathrm{s}}^{1/2}[Re^{-2}+0.014G_{\mathrm{s}}^2]^{1/4},$$

$$G_{\mathrm{s}} = \left|\frac{\mathrm{d}u}{\mathrm{d}y}\right|\frac{a}{\Delta u} \quad [20] \tag{6}$$

$$\frac{u(y)}{u^*} = \frac{1}{\kappa}\ln\left(1+\kappa\frac{yu^*}{v}\right)+c\left[1-\exp\left(-\frac{yu^*/v}{\chi}\right) - \frac{yu^*/v}{\chi}\exp\left(-0.33\frac{yu^*}{v}\right)\right]$$

$$\kappa = 0.4, \quad \chi = 11 \quad \text{and} \quad c = 7.4 \tag{7}$$

$$F_{\mathrm{b}} = \tfrac{4}{3}\pi a^3(\rho_{\mathrm{l}}-\rho_{\mathrm{v}})g. \tag{8}$$

In (5), $b = 1.0$ has been recommended for horizontal flow boiling of R113. In (3) and (6) $Re = 2\Delta ua/v$ where Δu is the relative velocity between the bubble center of mass and liquid. In (7) it has been recommended that $u^*/u_{\mathrm{l}} = 0.05$, where $u_{\mathrm{l}} = G(1-x)D/\rho_{\mathrm{l}}H$ is the mean liquid film velocity. Other symbols are described in the nomenclature. It is noted that in the absence of flow, (2) may be used to compute the pool boiling departure diameter.

Vapor bubble departure in pool boiling or lift-of in horizontal flow boiling occurs when the growth force is just balanced by the buoyancy force as described by equation (2). Due to the very small inertia of the bubble and very small contact area, the departure or lift-off occurs over a very short period of time or almost instantly comparing with the turbulent fluctuation and temperature fluctuation time scales. Hence, the departure or lift-off depends on the instantaneous Δu or ΔT. Since the departure diameter is a function of only one stochastic variable, wall superheat, it is relatively straightforward to relate the lift-off diameter pdf to that of the wall superheat

$$p_{\mathrm{L}}(d_{\mathrm{L}}) = p_{\Delta T}(\Delta T)\left|\frac{\delta\Delta T}{\delta d_{\mathrm{L}}}\right| \tag{9}$$

where d_{L} is the lift-off diameter, ΔT is the wall superheat, $p_{\mathrm{L}}(d_{\mathrm{L}})$ is the lift-off diameter pdf, and $p_{\Delta T}(\Delta T)$ is the wall superheat pdf. The derivative $\delta\Delta T/\delta d_{\mathrm{L}}$ is evaluated from equation (2).

Vapor bubble detachment diameters depend on two stochastic variables, liquid velocity and wall superheat and, thus, the prediction of the departure diameter pdf, $p_{\mathrm{d}}(d_{\mathrm{d}})$, is considerably more complex than predicting the lift-off diameter pdf, $p_{\mathrm{L}}(d_{\mathrm{L}})$. It is first necessary to know the variation of the departure diameter with variations in the instantaneous liquid velocity, u, and wall superheat, ΔT, which can be expressed as $d_{\mathrm{d}} = s(u,\Delta T)$ where $s(u,\Delta T)$ describes a surface in $(d_{\mathrm{d}},u,\Delta T)$ space. Since an explicit expression for d_{d} is not available, $s(u,\Delta T)$ must be determined from equation (1). A typical $s(u,\Delta T)$ surface is shown in Fig. 2 for horizontal flow boiling of refrigerant R113 at a saturation temperature of 60°C. In Fig. 2 u ranges from 0 to 4 m s^{-1} and ΔT ranges from 5 to 30°C, which is sufficient to cover the parameter space considered in this work. It is observed that d_{d} decreases with increasing u and increases with increasing ΔT, which is consistent with experimental observations. Defining ΔR as the region in the $(u,\Delta T)$ plane for which $d_{\mathrm{d}} < s(u,\Delta T) \leqslant d_{\mathrm{d}}+\delta d_{\mathrm{d}}$, the departure diameter pdf may be computed from

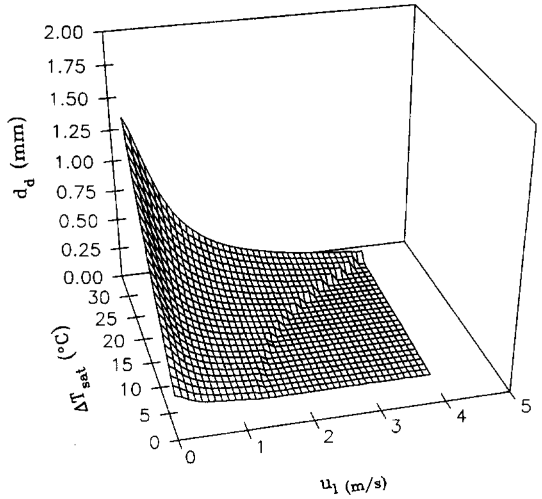

Fig. 2. Departure diameter surface $g(u,\Delta T)$ in $(d_{\mathrm{d}},u,\Delta T)$ space for horizontal flow boiling of refrigerant R113 at $T_{\mathrm{sat}} = 60$°C.

$$p_{\mathrm{d}}(d_{\mathrm{d}})\delta d_{\mathrm{d}} = \int_{\Delta R}\int p(u,\Delta T)\delta u\delta\Delta T \tag{10}$$

where $p(u,\Delta T)$ is the joint probability density function for liquid velocity and wall superheat. Assuming that statistical variations in liquid velocity and wall superheat are independent (10) may be expressed as,

$$p_{\mathrm{d}}(d_{\mathrm{d}})\delta d_{\mathrm{d}} = \int_{\Delta R}\int p_{\mathrm{u}}(u)p_{\Delta T}(\Delta T)\delta u\delta\Delta T \tag{11}$$

where $p_{\mathrm{u}}(u)$ is the liquid velocity pdf. In this work $p_{\mathrm{d}}(d_{\mathrm{d}})$ is numerically computed by approximating δd_{d} by $\Delta d_{\mathrm{d}} = 0.01$ mm and employing a search algorithm to determine ΔR such that $d_{\mathrm{d}} < s(u,\Delta T) \leqslant d_{\mathrm{d}}+\Delta d_{\mathrm{d}}$. The double integral in (11) is evaluated using the trapezoidal rule. The increment Δd_{d} is chosen such that it is much less than the mean departure diameter and larger than the increment $\Delta s(u,\Delta T)$. Since statistical variations in u are primarily due to turbulence and variations in ΔT are primarily due to distributions in nucleation site density and energy depletion from surrounding nucleation sites, the assumption of statistical independence of u and ΔT is justified.

3. COMPARISON WITH EXPERIMENTAL DATA

In order to implement the pdf models for bubble departure and lift-off given by equations (9) and (11), it is necessary to specify pdfs for wall superheat and turbulent velocity fluctuations in the liquid film. The first consideration will be given to statistical variations in wall superheat. Using a liquid crystal thermography technique, Kenning [15] measured statistical variations in wall superheat for saturated pool boiling with water at a heat flux of 100 kW m^{-2}. The data were presented as histograms. It is not unreasonable to fit a Gaussian distribution to these data. The ratios of the standard deviation of wall superheat to the mean wall superheat for the three tests are 0.24, 0.34 and 0.21. Kenning [16] also used liquid crystal thermography to measure statistical variations of wall superheat for subcooled pool boiling with water at heat fluxes of 50, 100, 150 and 200 kW m^{-2}. The corresponding ratios of standard deviation to mean wall superheat are 0.18, 0.15, 0.23 and 0.26. A Gaussian fit to these data also appears to be reasonable. Zeng and Klausner [21] reported using liquid crystal thermography to study nonuniformities in wall superheat associated with horizontal flow boiling. Although detailed quantitative evaluations of the data were never made, some qualitative observations are useful. The wall superheat demonstrated variations throughout the heating surface which were apparently randomly distributed and appeared to have Gaussian features. It is also worth noting that the ratio of the maximum variation in wall superheat to the mean wall superheat is roughly half that observed by Kenning [15, 16] for pool boiling. This observation may be explained by the supposition that the heat transfer associated with flow boiling is due to two additive components: that due to forced convection and that due to ebullition. The component due to forced convection tends to promote spatially uniform cooling, in contrast to that due to ebullition which is responsible for highly localized cooling. Therefore, it is to be expected that the ratio of the standard deviation of wall superheat to the mean wall superheat will be less for flow boiling than for pool boiling. Assuming that the statistical variations of wall superheat follow a Gaussian distribution, the associated pdf may be modeled for the present purpose as

$$p_{\Delta T}(\Delta T) = \frac{1}{\sqrt{2\pi}\sigma_{\Delta T}} e^{-1/2((\Delta T-\Delta T_{\mathrm{m}})/\sigma_{\Delta T})^2} \qquad (12)$$

where ΔT_{m} is the mean wall superheat and $\sigma_{\Delta T}$ is the standard deviation of wall superheat.

Fluctuations of liquid velocity in flow boiling may be due to interfacial waves as well as bulk turbulence, both of which are stochastic in time and space. Detailed experimental investigations of liquid film velocity fluctuations in separated two-phase flows are not available. For the sake of simplicity and lack of experimental evidence, the pdf of the liquid velocity at the bubble center of mass is assumed to be Gaussian

$$p_{\mathrm{u}}(u) = \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{u}}} e^{-1/2((u-u_{\mathrm{m}})/\sigma_{\mathrm{u}})^2} \qquad (13)$$

where u_{m} is the mean liquid velocity and σ_{u} is the standard deviation of the liquid velocity.

Experimental data describing the statistical distribution of bubble detachment diameters are typically presented in the form of histograms. These data describe the probability that $(d-\Delta d/2 < d \leqslant d+\Delta d/2)$ where Δd is the diameter increment used in constructing the histograms. In order to compare the present model with experimental data, the probability of finding a bubble detachment diameter within a specified increment is computed from

$$P\left(d-\frac{\Delta d}{2} < d \leqslant d+\frac{\Delta d}{2}\right) = \int_{d-\Delta d/2}^{d+\Delta d/2} p(\zeta)\delta\zeta \qquad (14)$$

where $p(\zeta)$ is the pdf for either vapor bubble departure or lift-off.

Comparisons are first made with horizontal flow boiling lift-off data reported by Zeng *et al.* [13] and Bernhard [14]. A comparison between the measured probabilities that $(d-\Delta d/2 < d \leqslant d+\Delta d/2)$ and those computed for a given lift-off diameter are shown in Fig. 3 for a mean wall superheat ranging from 8.3 to 11.6°C and mean liquid velocity ranging from 0.37 to 0.75 m s^{-1}. In computing the pdf of lift-off diameter equation (9) is used in conjunction with equation (12), where it has been assumed that $\sigma_{\Delta T} \approx 1/8\Delta T_{\mathrm{m}}$ so that the predicted probabilities best fit those measured. This assumption is consistent with the liquid crystal thermography measurements previously discussed. Both the mean lift-off diameter and standard deviation increase with increasing wall superheat. For the wall superheats considered herein, the model prediction is in very good agreement with the experimental data.

The experimental horizontal flow boiling departure data reported by Klausner *et al.* [12] and Bernhard [14] are considered next. Figure 4 compares the measured and computed probabilities for departure diameter. In these data, the mean liquid velocity ranges from 0.38 to 0.76 m s^{-1} while the wall superheat varies from 13.6 to 16.4°C. Equations (12) and (13) were used in conjunction with equation (11) to predict the pdf of departure diameter. In using equation (13) it was assumed again that $\sigma_{\Delta T} \approx 1/8\Delta T_{\mathrm{m}}$. In the bubble detachment diameter model proposed by Zeng *et al.* [13], good results are obtained by assuming the liquid film friction velocity is 5% of the mean liquid velocity. Since no other information is available on liquid film turbulence, it is assumed that $\sigma_{\mathrm{u}} \approx 0.05u_{\mathrm{m}}$. It is seen that the mean departure diameter declines with increasing mean liquid velocity and decreasing wall superheat. Again, good agreement with the data is demonstrated. Figure 5 compares the measured and computed probabilities of departure diameter in which the mean liquid velocity does not vary appreciably and mean wall superheat varies from 13.8 to 16.3°C. Good agreement in the trend between the measured and predicted probabilities is demonstrated.

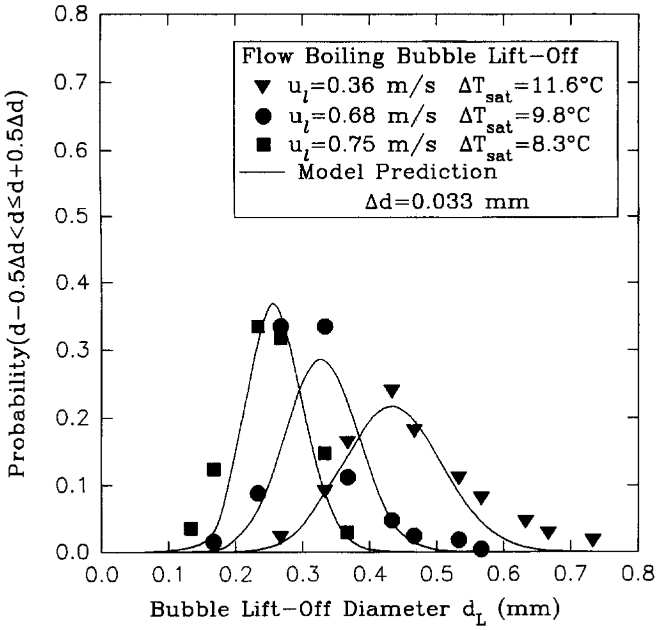

Fig. 3. Statistical distribution of vapor bubble lift-off diameters in horizontal flow boiling.

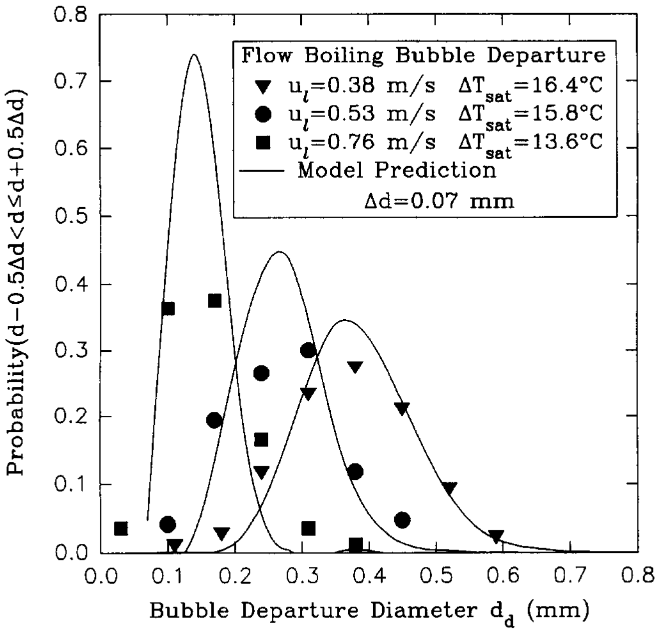

Fig. 4. Statistical distribution of vapor bubble departure diameters in horizontal flow boiling with variable liquid velocity.

4. DISCUSSION

The present modeling has successfully connected the pdf of the vapor bubble detachment diameter to those of the wall superheat and mean liquid velocity. Although direct experimental evidence for the exact behavior of the wall superheat and mean liquid velocity pdfs are not available, physical justifications for their assumed behavior have been made. The objective of this paper has been to establish a framework for understanding and predicting the stochastic features of vapor bubble detachment in boiling systems. Over a limited range of boiling conditions the proposed mechanism and model is successful. In order for such a model to be incorporated into heat transfer predictions, further investigations are required to better understand the stochastic variations of liquid velocity and wall superheat in boiling systems. Although an assumed Gaussian distribution for wall superheat variations has been demonstrated to be adequate, it remains to be demonstrated that the liquid velocity fluctuations are adequately represented by a Gaussian

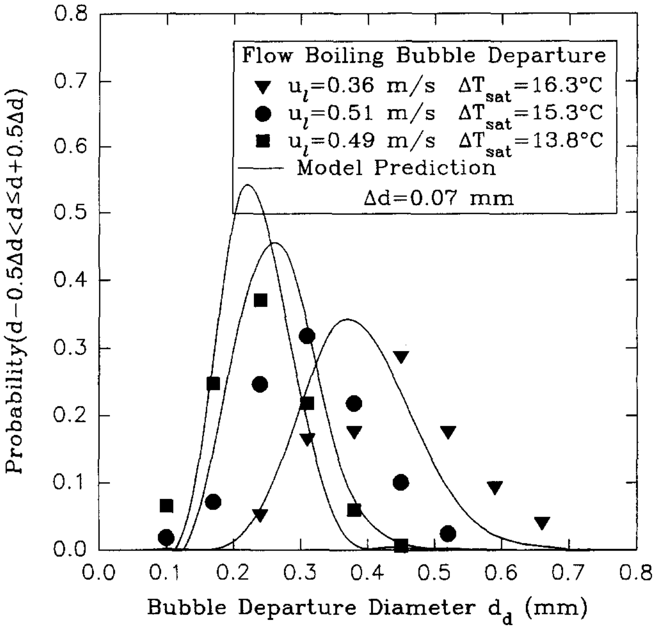

Fig. 5. Statistical distribution of vapor bubble departure diameters in horizontal flow boiling with variable wall superheat.

distribution. If so, how do the standard deviations of wall superheat and liquid velocity depend on flow and thermal conditions of the boiling system? The resolution of these issues will require further experimental investigations.

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